

Forms [14 CM] and [43 W] through [43 AC]

[14 CM] Kolany's Patching Lemma. Assume that $\{A_j : j \in J\}$ is a family of non-empty sets and $\{\mathcal{F}_j : j \in J\}$ is a family of non-empty sets of functions such that for every $j \in J$ and every $f \in \mathcal{F}_j$, $\text{dom } f = A_j$. Assume that for every finite $J_0 \subseteq J$ there is a function F_0 such that for all $j \in J_0$, $F_0 \upharpoonright A_j \in \mathcal{F}_j$, then there exists a function F such that for all $j \in J$, $F \upharpoonright A_j \in \mathcal{F}_j$. Kolany [1999].

[43 W] Countable products of compact Hausdorff spaces are Baire. Brunner [1983c] and note 28.

[43 X] Products of compact Hausdorff spaces are Baire. Herrlich/Keremedis [1998] and note 28.

[43 Y] Products of pseudo-compact spaces are Baire. Herrlich/Keremedis [1998] and note 28.

[43 Z] Products of countably compact, regular spaces are Baire. Herrlich/Keremedis [1998] and note 28.

[43 AA] Products of regular-closed spaces are Baire. Herrlich/Keremedis [1998] and note 28.

[43 AB] Products of Čech-complete spaces are Baire. Herrlich/Keremedis [1998] and note 28.

[43 AC] Products of pseudo-complete spaces are Baire. Herrlich/Keremedis [1998] and note 28.

In note 28 replace the first definition with the following

Definition. Let (X, T) be a topological space.

1. A set $Y \subseteq X$ is *nowhere dense* if the closure of Y has empty interior.
2. A set $Y \subseteq X$ is *meager* or *of the first category* if Y is a countable union of nowhere dense sets.
3. A set $Y \subseteq X$ is *perfect* if Y is closed, non-empty and has no isolated points.
4. A set $Y \subseteq X$ has the *Baire property* if $(Y \setminus U) \cup (U \setminus Y)$ is meager for some open set U .
5. (X, T) is *regular* if points are closed and every neighborhood of a point contains a closed neighborhood of that point.
6. A *regular filter* on (X, T) is a subset F of $\mathcal{P}(X)$ which is closed under intersections and supersets such that for some collection \mathcal{U} of open sets \mathcal{U} generates F (That is, $F = \{y \subseteq X : \text{for some finite subset } \mathcal{U}_0 \text{ of } \mathcal{U}, \bigcap \mathcal{U}_0 \subseteq y\}$.) and for some collection \mathcal{C} of closed sets \mathcal{C} generates F .
7. (X, T) is *regular-closed* if (X, T) is regular and any regular filter on X has a non-empty intersection.
8. (X, T) is *Baire* or is a *Baire space* if the intersection of each countable sequence of dense, open sets in X is dense in X .
9. (X, T) is *sequentially complete* if every sequence has a convergent subsequence.
10. (X, T) is *Čech complete-I* if it satisfies
 - (i) (X, T) is regular (every neighborhood of a point $x \in S$ contains a closed neighborhood of x) and
 - (ii) there is a denumerable collection $\{\mathcal{C}_n : n \in \omega\}$ of open covers of X such that for any collection F of closed subsets of X with the finite intersection property, if for each n , F contains a subset of diameter less than \mathcal{C}_n (that is, $(\exists u \in F)(\exists c \in \mathcal{C}_n)(u \subseteq c)$), then F has a non-empty intersection.

(This is the definition of Čech complete used in form [43 E])

11. (X, T) is *Čech complete-II* if (X, T) is homeomorphic to a G_δ set in a compact, T_2 space. ($G_\delta \equiv$ countable intersection of open sets.)
(This is the definition of Čech complete used in *K9* below. *K9* is equivalent to form 106.)
12. A collection \mathcal{B} of non-empty open sets is called a *regular pseudo-base* for (X, T) if:
 - (i) For each non-empty $A \in T$, there is some $B \in \mathcal{B}$ such that $clB \subseteq A$ and
 - (ii) If $A \neq \emptyset$ is in T and $A \subseteq B$ for some $B \in \mathcal{B}$, then $A \in \mathcal{B}$.
13. (X, T) is *pseudo-complete* provided there is a sequence $(\mathcal{B}_n)_{n \in \omega}$ of regular pseudo-bases such that for every regular filter F on X , if F has a countable base and meets each \mathcal{B}_n then F has non-empty intersection.
14. (X, T) is *co-compact* if \exists a family F of closed sets such that
 - (i) If $G \subseteq F$ and G has the finite intersection property, then $\bigcap G \neq \emptyset$ and
 - (ii) If $x \in \mathcal{O}$, with \mathcal{O} open, then $\exists A \in F$ with $x \in A^\circ$ (the interior of A) and $A \subseteq \text{closure}(\mathcal{O})$.
15. (X, T) is *pseudo-compact* if every continuous real valued function on (X, T) is bounded.
16. (X, T) is *scattered* or *Boolean* if it is Hausdorff and has a basis of clopen sets.