Professor Kyriakos Keremedis has informed us of recent results showing that forms 11, 12, 73, 75, 92, 337, 338, 339, and 360 are all equivalent to form 94. He has also pointed out some new equivalents of form 94. This requires the following changes in the *Consequences of the Axiom of Choice* Project:

1. **Additions to Part VI, the Bibliography**

Herrlich, H.Strecker, E.

[**1997**] *When is* $\mathbb{N}$ *Lindelöf*, Comment. Math. Univ. Carolinae **38**, 553-556.

Keremedis, K.

[**1998**] *Countable disjoint unions and weak forms of AC*, preprint.

2. **Changes in Part II, the Topical List of Forms**

• Make each of the changes below:

The old form 11 should be changed to

[**94 C**] $\mathbb{R}$ is hereditarily Lindelöf. Lindelöf [**1905**] and note 40.

form 12 should be changed to

[**94 D**] $\mathbb{R}$ is Lindelöf. Herrlich/Strecker [**1997**], Young [**1903**],and note 40.

form 360 should be changed to

[**94 E**] Every second countable topological space is Lindelöf. Herrlich/Strecker [**1997**], G. Moore [**1982**] p 236 and note 40.

form 73 should be changed to

[**94 F**] For every $A \subseteq \mathbb{R}$ and $x \in \mathbb{R}$ the following definitions are equivalent:
(1) x is in the closure of A iff every neighborhood of x intersects A.
(2) x is in the closure of A iff there is a sequence $\{x_n\} \subseteq A$ such that $\lim x_n = x$.
Herrlich/Strecker [**1997**], Jech [**1973b**] p 21, and note 40.

form 92 should be changed to

[**94 G**] Every subset of $\mathbb{R}$ is separable. Herrlich/Strecker [**1997**], Jech [**1973b**] p 142, and note 40.

form 75 should be changed to

[**94 H**] Every subspace of a separable metric space is separable. Jech [**1973b**] p 21 and note 40.

form 337 should be changed to

[**94 I**] Every separable metric space is Lindelöf. Good/Tree [**1995**] and note 40.

form 338 should be changed to

[**94 J**] Every second countable metric space is Lindelöf. Good/Tree [**1995**] and note 40.

form 339 should be changed to

[**94 K**] Every second countable metric space is separable. Good/Tree [**1995**] and Keremedis [**1998**].

form [73 A] should be changed to

[**94 O**] For all $A \subseteq \mathbb{R}$, $x \in A$ and $f : A \to \mathbb{R}$ the following are equivalent:
(1) $(\forall \epsilon > 0)(\exists \delta > 0)(\forall y \in A)(|y - x| < \delta$ implies $|f(y) - f(x)| < \epsilon)$
(2) Whenever $\{x_n\} \subseteq A$ and $\lim x_n = x$ then $\lim f(x_n) = f(x)$.
Herrlich/Strecker [**1997**] and note 5.

form [73 B] should be changed to

[94 P]$(\forall f:\mathbb{R}\to\mathbb{R})(\forall x\in\mathbb{R})$ the following are equivalent:
(1) $(\forall\epsilon>0)(\exists\delta>0)((\forall y\in\mathbb{R})(|y-x|<\delta\to|f(y)-f(x)|<\epsilon)$
Herrlich/Strecker [**1997**].

- Add the following new forms:

  [94 L] $\mathbb{Q}$ is Lindelöf. Herrlich/Strecker [**1997**]. Put [94 L] in TOPOLOGICAL FORMS III. General Topology

  [94 M] In $\mathbb{R}$ every unbounded set contains a countable unbounded set. Herrlich/Strecker [**1997**]. Put [94 M] in TOPOLOGICAL FORMS VI. Properties of $\mathbb{R}$

  [94 N] Partial Choice for Countable Families of Sets of Reals: Every countable family of non-empty sets of real numbers has an infinite subset with a choice function. Sierpinski [**1916**]. Put [94 N] in CHOICE FORMS IV. Partial Choice PC

  [13 C] Strong Bolzano-Weierstrass Theorem: In $\mathbb{R}$ for every bounded, infinite set A, there is a convergent, injective sequence whose terms are in A. Herrlich/Strecker [**1997**]. Put [13 A] in TOPOLOGICAL FORMS VI. Properties of $\mathbb{R}$

3. **Changes in Part I, Numerical List of Forms**

Delete forms 11, 12, 73, 75, 92, 337, 338, 339, and 360. Add the equivalents of form 94 listed above and the equivalent of form 13.

4. **Changes in Part III, Models**

Delete all references to forms 11, 12, 73, 92, 337, 338, 339, and 360. In the Cohen models section only 337, 338, and 339 occur and they only occur in the description of M1. In the Fraenkel-Mostowski models section 12 occurs only in the introduction. 11, 73, and 92 occur in the introduction and in the descriptions of most of the FM models.

5. **Changes in Part IV, Notes**

1. Eliminate notes 4, 6 and 10. (The results are in Herrlich/Strecker [1997].)
2. Replace note 5 with and its table of contents entry with:

5. A proof that [94 O] implies 74

NOTE 5

[94 O] $\to$ form 74. [94 O] is For all $A\subseteq\mathbb{R}$, $x\in A$ and $f:A\to\mathbb{R}$ the following are equivalent:
(1) $(\forall\epsilon>0)(\exists\delta>0)(\forall y\in A)(|y-x|<\delta$ implies $|f(y)-f(x)|<\epsilon)$
(2) Whenever $\{x_n\}\subseteq A$ and $\lim x_n=x$ then $\lim f(x_n)=f(x)$.
and form 74 is: For every $A\subseteq\mathbb{R}$ the following are equivalent:
(1) A is closed and bounded.
(2) Every sequence $\{x_n\}\subseteq A$ has a convergent subsequence with limit in A.)

Proof. Assume [94 O]. The usual proof that part (1) of form 74 implies part (2) doesn't require any choice. So assume $A\subseteq\mathbb{R}$ and A satisfies (2) of 74 but not (1) of 74. Assume first that A is bounded. Then A is not closed so there is an $x_0\notin A$ such that every neighborhood of x_0 intersects A. Define $f:A\cup\{x_0\}\to\mathbb{R}$ by $f(x)=1$ if $x\in A$ and $f(x)=0$ if $x=x_0$. Then (1) of [94 O] fails (with

$\epsilon = 1$) so there is a sequence $\{x_n\} \subseteq A \cup \{x_0\}$ such that $\lim f(x_n) \neq f(x_0)$. This means we can assume $\{x_n\} \subseteq A$ (replacing $\{x_n\}$ by a subsequence if necessary). By (2) of form 74, $\lim x_n \in A$ that is $x_0 \in A$, a contradiction. Now assume that A is unbounded. Let f be a monotone, increasing function from the interval $(-1, 1)$ one to one, onto $\mathbb{R}$ which satisfies $(\forall a, b \in (-1, 1))(|a - b| \leq |f(a) - f(b)|)$. (For example, $f(x) = \frac{2x}{1-x^2}$ would work.) Then $f^{-1}(A)$ is bounded and satisfies (2) of 74 because if $\{x_n\}$ is a sequence in $f^{-1}(A)$, then $\{f(x_n)\}$ is a sequence in A. Since A satisfies (2) of 74, $\{f(x_n)\}$ has a convergent subsequence with limit in A. Using the property which we required f to have we infer that f^{-1} applied to the convergent subsequence mentioned above gives a convergent subsequence of $\{x_n\}$ with limit in $f^{-1}(A)$. Since we have proved (2) of 74 implies (1) of 74 for bounded sets we conclude that $f^{-1}(A)$ is closed. Hence there are numbers a and b such that $-1 < a < b < 1$ and $(\forall x \in f^{-1}(A))(a < x < b)$. Since f is monotone increasing $(\forall x \in A)(f(a) < x < f(b))$. Hence A is bounded. □

3. Replace note 40 and its table of contents entry by the following:
40. The equivalents of form 94

NOTE 40

In this note we consider the equivalents of form 94. Brunner [**1982d**] proves that [94 B] implies form 13 (lemma 6, p. 164) by showing that the negation of form 13 implies the negation of [94 A] which implies the negation of [94 B]. Since [94 A] clearly implies [94 B], it only remains to prove that [94 A] implies 94 (to complete the argument for the equivalence of 94, [94 A] and [94 B]). Herrlich/Strecker [**1997**] have shown that forms 94 and [94 A] are equivalent. In addition they show that forms [94 A], [94 D], [94 E], [94 F], [94 G], [94 L], [94 M], [94 O] and [94 P] are equivalent. It is clear that [94 E] implies [94 C] implies [94 D]. This answers a question of G. Moore [**1982**] p 322: Does [94 D] imply [94 C]? G. Moore also asks if form 13 implies [94 D]. This is also answered in the negative since 13 is true and 94 false in $\mathcal{M}6$.

It is shown in Keremedis [**1998**] that [94 K] is equivalent to 94 and Sierpinski [**1916**] shows the equivalence of [94 N] and [94].

To get equivalence of [94 H], [94 I], and [94 J] to form 94, we note first that (in ZF^0) every separable metric space is second countable. From this it follows that [94 K] implies [94 H] and [94 J] implies [94 I]. Combining these with the following easy implications gives us the desired equivalences. [94 K] $\rightarrow$ [94 H], [94 E] $\rightarrow$ [94 J], and [94 I] $\rightarrow$ [94 D]. Most of the above facts were pointed out to us by K. Keremedis.

4. In note 18

A . Eliminate 11, 12, 73, and 92 from list 1: Forms that are transferable.

B . Eliminate 11, 12, 73, and 92 from list 3: Forms that are true in every permutation model. And in the parenthetical remark say "Each one of these 49 forms is implied by one of the 12 forms 6, 37, 91, 130, 273, 305, 309, 313, 361, 363, 368, 369.)" (I've just eliminated 11 and 92 from the original list.)

5. In note 103 in the list following " Similarly, the following forms are boundable and therefore transferable:" eliminate forms 11, 12, 73 and 92

6. **Changes in V, Reference for Table 1**

eliminate the entry at positions n,k where either n or k is 11, 12, 73, 75, 92, 337, 338, 339, or 360. Add

94 74 (1) note 5