Changes in the lists of forms

Add the following forms:

1. **FORM 28(p)** (Where p is a prime) AL20($\mathbb{Z}_p$): Every vector space V over $\mathbb{Z}_p$ has the property that every linearly independent subset can be extended to a basis. ($\mathbb{Z}_p$ is the p element field.) Bleicher [**1964**], Rubin, H.Rubin, J. [**1985, p.119, AL20**]. Put in ALGEBRAIC forms, V. Vector spaces

2. **FORM 239** AL20($\mathbb{Q}$): Every vector V space over $\mathbb{Q}$ has the property that every linearly independent subset of V can be extended to a basis. Bleicher [**1964**], Rubin, H.Rubin, J. [**1985, p.119, AL20**]. Put in ALGEBRAIC forms, V. Vector spaces

3. **FORM 427** $\exists F$ AL20(F): There is a field F such that every vector space V over F has the property that every independent subset of V can be extended to a basis. Bleicher [**1964**], Rubin, H.Rubin, J. [**1985, p.119, AL20**]. Put in ALGEBRAIC forms, V. Vector spaces

4. **FORM 428** $\exists F$ B(F): There is a field F such that every vector space over F has a basis. Bleicher [**1964**], Rubin, H.Rubin, J. [**1985, p.119, B**]. Put in ALGEBRAIC forms, V. Vector spaces

5. **FORM 429(p)** (Where p is a prime) B: Every vector space over $\mathbb{Z}_p$ has a basis. ($\mathbb{Z}_p$ is the p element field.) Bleicher [**1964**], Rubin, H.Rubin, J. [**1985, p.119, B**]. Put in ALGEBRAIC forms, V. Vector spaces

6. **FORM 430(p)** (Where p is a prime) AL21(p): Every vector space over $\mathbb{Z}_p$ has the property that for every subspace S of V, there is a subspace S' of V such that $S \cap S' = \{0\}$ and $S \cup S'$ generates V in other words such that $V = S \oplus S'$. Bleicher [**1964**], Rubin, H.Rubin, J. [**1985, p.119, AL21**]. Put in ALGEBRAIC forms, V. Vector spaces

7. [**430 A(p)**] (Where p is a prime) MC(∞, ∞, relatively prime to p), MC4(p): For every set X of non-empty sets there is a function f with domain X such that $\forall u \in X$, $f(u)$ is a finite non-empty subset of u such that $|f(u)|$ and p are relatively prime. See form 218 and Bleicher [**1964**], Rubin, H.Rubin, J. [**1985, p.124, theorems 6.37 and 6.38**]. Put in CHOICE FORMS, III. mc

8. [**218 B**] For all primes p, MC(∞, ∞, relatively prime to p), $\forall p$, MC4(p): For every set X of non-empty sets there is a function f with domain X such that $\forall u \in X$, $f(u)$ is a finite non-empty subset of u such that $|f(u)|$ and p are relatively prime. See form 218, Bleicher [**1964**], Rubin, H.Rubin, J. [**1985, p.124**] and note 161. Put in CHOICE FORMS, III. mc

9. Add to form [218 A]: See note 161.

10. **[67 AD]** $\exists F$ AL21(F): There is a field F such that every vector space over F has the property that for every subspace S of V, there is a subspace S' of V such that $S \cap S' = \{0\}$ and $S \cup S'$ generates V in other words such that $V = S \oplus S'$. Bleicher **[1964]**, Rubin, H.Rubin, J. **[1985, pp. 122,123, theorems 6.35 and 6.36]**. Put in ALGEBRAIC forms, V. Vector spaces

11. **[67 AE]** $\exists F$ of characteristic 0 such that AL21(F): There is a field F of characteristic 0 such that every vector space over F has the property that for every subspace S of V, there is a subspace S' of V such that $S \cap S' = \{0\}$ and $S \cup S'$ generates V in other words such that $V = S \oplus S'$. Bleicher **[1964]**, Rubin, H.Rubin, J. **[1985, pp. 122,123, theorems 6.35 and 6.36]**. Put in ALGEBRAIC forms, V. Vector spaces

12. **[67 AF]** AL21($\mathbb{Q}$): Every vector space over $\mathbb{Q}$ has the property that for every subspace S of V, there is a subspace S' of V such that $S \cap S' = \{0\}$ and $S \cup S'$ generates V in other words such that $V = S \oplus S'$. Bleicher **[1964]**, Rubin, H.Rubin, J. **[1985, pp. 122,123, theorems 6.35 and 6.36]**. Put in ALGEBRAIC forms, V. Vector spaces

Changes in the Notes

New note

NOTE 161

In Rubin, H.Rubin, J. **[1985]** pages 122–124, theorems 6.35 through 6.37 it is shown that forms [218 A] and [218 B] are equivalent. It is clear that Form 218 ($\forall n \in \mathbb{N} - \{0,1\}$,MC$(\infty, \infty$, relatively prime to n)) implies [218 B] (For all prime numbers p, MC$(\infty, \infty$, relatively prime to p)) . In this note we argue that [218 B] implies Form 218. Assume that $n \in \mathbb{N} - \{0,1\}$ and that X is a family of non-empty sets. Let $X' = X \cup \{y : y$ is finite and non-empty and $\exists z \in X$ such that $y \subseteq z\}$. Let $\{p_i : 1 \leq i \leq k\}$ be the set of prime divisors of n. By [218 B] there are functions f_i, $1 \leq i \leq k$ such that for $1 \leq i \leq k$, f_i is a function with domain X' and for all $y \in X'$, $f_i(y)$ is a finite non-empty subset of y such that $|f_i(y)|$ and p_i are relatively prime. Let $f = f_1 \circ f_2 \circ \cdots \circ f_k$ then for $y \in X$, one of the sets $f(y), f(f(y)) = f^2(y), f^3(y), \ldots$ will have cardinality relatively prime to p_i for all i, $1 \leq i \leq k$ and therefore will have cardinality relatively prime to n. Define $g(y) = f^r(y)$ where r is the least natural number for which $|f^r(y)|$ is relatively prime to n. The function g is the function required by the conclusion of Form 218.

Changes in the Models Chapter

Add form 110 to the list of forms false in N2*(3) and in the description add

Form 110 and 346 are false in this model since it is shown in Keremedis [**2001a**], theorem 7 that each of forms 110 and 346 imply form 373(n) for all $n \in \mathbb{N} - \{0, 1\}$ and form 373(3) is clearly false in $\mathcal{N}2^*(3)$.

Changes in References for Relationships between forms

These are the lines that have been added to the file rfb1.tex the most recent version or rfb1.tex is available on the "Changes and Additions" page.

109	28	1	clear
109	66	1	clear
427	67	1	clear, see [67 AD]
430	67	1	clear
66	110	1	clear
88	110	3	Keremedis [**2001a**] (N2*(3)T)
111	110	3	Keremedis [**2001a**] (N2*(3)T)
141	110	5	Keremedis [**2001a**] (N2*(3))
239	110	1	clear
333	110	5	Keremedis [**2001a**] (N2*(3))
109	218	1	clear
109	239	1	clear
28	427	1	clear
239	427	1	clear
66	428	1	clear
110	428	1	clear
427	428	1	clear
429	428	1	clear
28	429	1	clear
28	430	1	clear
218	430	1	clear