

## Additions to Part I: Numerical List of Forms

Eliminate [8 F] and 360 from the list of unused forms in Parts I and II and add the following list of new forms.

[0 AN] No maximal open free filter in a  $T_2$  topological space has a countable filter base. Herrlich/Tachtsis [1999c] and note 10.

[0 AO] Countably compact pseudometric spaces are Baire. Herrlich/Keremedis [1999a], notes 10 and 28.

[0 AP] Using the discrete topology on  $2$ ,  $2^m$  is compact for every well ordered cardinal number  $m$ . Keremedis [1999b].

[1 CI] Every compact space is A-U compact. Herrlich [1996a], Howard [1990], and note 6.

[1 CJ] Every B compact Hausdorff space is A-U compact. Herrlich [1996a] and note 6.

[1 CK] Products of A-U compact (Hausdorff) spaces are A-U compact. Herrlich [1996a] and note 6.

[1 CL] Finite products of A-U compact spaces are A-U compact. Herrlich [1996a] and note 6.

[1 CM] Spaces with finite topologies are A-U compact. Herrlich [1996a] and note 6.

[1 CN] Products of spaces with finite topologies are compact. Alas [1969].

[1 CO(X,Y)] The product of  $X$  spaces is  $Y$ , where  $X$  is either “compact”, “S-B compact”, or “linearly compact” and  $Y$  is either “compact” or “S-B compact”. Howard [1990] and note 6.

[1 CP] Form 384 + Form 8. Herrlich/Steprans [1997].

[1 CQ] Closed Filter Extendability for  $T_0$  Spaces. Every closed filter in a  $T_0$  topological space can be extended to a maximal closed filter. Keremedis/Tachtsis [1999c] and note 10.

[1 CR] The Ascoli Theorem for A-U compactness. See form [14 CU], note 6 and Herrlich [1997c]

[1 CS] Every W compact pseudometric space is A-U compact. See Howard [1990] (the proof of Theorem 3) and note 6.

[1 CT] Every sequentially compact pseudometric space is A-U compact. Herrlich [1997a]

[1 CU] Products of topological spaces with finite topologies are A-U compact. Herrlich [1997a] and note 6.

[1 CV] Products of finite discrete topological spaces are A-U compact. Herrlich [1996a] and note 6.

[1 CW] Every closed (open) filter in a  $T_1$  topological space extends to a maximal closed (open) filter with a well orderable filter base. Keremedis/Tachtsis [1999a].

[1 **CX**] For every set  $A \neq \emptyset$ , every filter  $\mathcal{F} \subseteq \mathcal{P}(A)$  extends to an ultra filter with a well orderable filter base. Keremedis/Tachtsis [1999a].

[1 **CY**] Every partial function on a set  $A$  can be defined as follow: Let  $\varrho, \sigma, \varphi, \tau_1, \tau_2, \tau_3$  be equivalence relations on  $A$  such that  $\sigma$  has a distinguished equivalence class  $D$  which is a set of distinct representatives for the equivalence classes of  $\varrho$ ,  $\varphi$  has at most three equivalence classes  $D_1, D_2$ , and  $D_3$ , and the equivalence classes of  $\tau_n, n = 1, 2, 3$ , have at most two elements. If  $x \in A$  has a  $\varrho$ -representative  $u \in D$  and there is an  $n = 1, 2, 3$  such that  $u \in D_n$ , then choose the smallest  $n$  with this property and define  $f(x) = u$ , where  $\{u, f(x)\} \in D_n$ . Armbrust [1986].

[8 **F**] Every pseudometric space with a countable base is separable. Bentley/Herrlich [1998] and note 10.

[8 **V**] Every pseudometric Lindelöf space is separable. Bentley/Herrlich [1998] and note 10.

[8 **W**] Subspaces of pseudometric spaces are separable. Bentley/Herrlich [1998] and note 10.

[8 **X**] Every sequentially bounded pseudometric space is totally bounded. Bentley/Herrlich [1998] and note 10.

[8 **Y**] Every totally bounded pseudometric space is separable. Bentley/Herrlich [1998] and note 10.

[8 **Z**] Every sequentially bounded pseudometric space is separable. Bentley/Herrlich [1998] and note 10.

[8 **AA**] Baire Category Theorem for Complete Pseudometric Spaces with a countable base. Every non-empty complete pseudometric space with a countable base is of the second category (non-meager). Bentley/Herrlich [1998] and note 28.

[8 **AB**] Baire Category Theorem for Complete Totally Bounded Pseudometric Spaces. Every non-empty complete totally bounded pseudometric space is of the second category (non-meager). Bentley/Herrlich [1998], and notes 6 and 28.

[8 **AC**] Every sequentially compact pseudometric space is totally bounded. Bentley/Herrlich [1998] and notes 6 and 10.

[8 **AD**] Every totally bounded, complete pseudometric space is compact. Bentley/Herrlich [1998] and notes 6 and 28.

[8 **AE**] Every sequentially compact pseudometric space is compact. Bentley/Herrlich [1998] and note 10.

[8 **AF**] In a pseudometric space, every infinite subset has an accumulation point if and only if the space is complete and totally bounded. Bentley/Herrlich [1998] and note 10.

[8 **AG**] Compact pseudometric spaces are separable. Bentley/Herrlich [1998] and note 10.

[8 **AH**] In a metric space  $X$ , if  $x$  is an accumulation point of a subset  $A \subseteq X$  then there is a sequence of elements of  $A$  which converges to  $x$ . Herrlich/Steprans [1997].

- [8 **AI**] A function from one metric space to another is continuous if and only if it is sequentially continuous. Herrlich/Steprans [1997].
- [8 **AJ**] Countable products of pseudometric spaces are Baire. Herrlich/Keremedis [1999a], notes 10 and 28.
- [8 **AK**] Countable products of pseudometric spaces are compact. Herrlich/Keremedis [1999a] and note 10.
- [8 **AL**] For pseudometric spaces, every Cauchy filter converges if and only if every Cauchy sequence converges. Herrlich/Keremedis [1999a] and note 10.
- [8 **AM**] Every sequentially compact pseudometric space is Baire. Herrlich/Keremedis [1999a], notes 6, 10 and 28.
- [8 **AN**] Every sequentially compact, totally bounded pseudometric space is Baire. Herrlich/Keremedis [1999a], notes 6, 10 and 28.
- [9 **U**] There exists an A-U compact Hausdorff topology on every set. Herrlich [1996a] and note 6.
- [9 **V**] There exist an A-U compact  $T_1$  topology on every set. Herrlich [1996a] and note 6.
- [9 **W**] The Alexandroff compactification of a discrete space is A-U compact. Herrlich [1996a] and note 6.
- [9 **X**] In every sequentially compact pseudometric space, every infinite set has an accumulation point. Bentley/Herrlich [1998] and note 10.
- [9 **Y**] Every infinite tree has a countably infinite chain or a countably infinite antichain. Keremedis [1999a] and note 21.
- [10 **P**] Countable products of finite Hausdorff spaces are Baire. Herrlich/Keremedis [1999b] and note 28.
- [10 **Q**] Countable products of non-empty finite sets are non-empty. Herrlich/Keremedis [1999b].
- [10 **R**] For every sequence  $(X_n)_{n \in \omega}$  of non-empty finite sets,  $\mathcal{P}(\bigcup_{n \in \omega} X_n)$  is linearly orderable. Herrlich/Keremedis [1999b].
- [14 **CN**] A system of polynomial equations over the field  $\{0, 1\}$  has a solution if and only if every finite subsystem has a solution. Note 30.
- [14 **CO**] Every  $B$  compact space is compact. Herrlich [1996a] and note 6.
- [14 **CP**] Every closed filter in a topological space is contained in a prime closed filter. Herrlich/Steprans [1997] and note 10.
- [14 **CQ**] Every  $z$ -filter in a topological space is contained in a prime  $z$ -filter. Herrlich/Steprans [1997] and note 10.
- [14 **CR**] Every  $z$ -filter in a topological space is contained in a maximal  $z$ -filter. Herrlich/Steprans [1997] and note 10.

[14 CS] Every clopen filter in a topological space is contained in a prime (=maximal) clopen filter. Herrlich/Steprans [1997] and note 10.

[14 CT] Every open filter in a topological space is contained in a prime open filter. Herrlich/Steprans [1997] and note 10.

[14 CU] The Ascoli Theorem. If  $X$  is a locally compact Hausdorff space,  $Y$  is a metric space,  $C_\infty(X, Y)$  is the space of all continuous functions from  $X$  to  $Y$  with the compact-open topology, and  $F$  is a subspace of  $C_\infty(X, Y)$  then the following conditions are equivalent:

- (1)  $F$  is compact
- (2) (a) For each  $x \in X$ , the set  $F(x) = \{f(x) : f \in F\}$  is compact in  $Y$ .  
 (b)  $F$  is closed in the product space  $Y^X$ ,  
 (c)  $F$  is equicontinuous.

Herrlich [1997b] and note 10.

[14 CV] The Ascoli Theorem for B compactness. See form [14 CU], note 6, and Herrlich [1997b].

[14 CW] The projective limit of a directed projective system of non-empty, compact,  $T_2$  topological spaces is a non-empty, compact,  $T_2$  topological space. Rav [1976] and note 71. (Compare with [14 CD] through [14 CG].)

[14 CX] The projective limit of a directed projective system of finite topological spaces with the discrete topology is a non-empty, compact,  $T_2$  topological space. Rav [1976] and note 71. (Compare with [14 CD] through [14 CG].)

[14 CY] Let

$$(*) \quad G_i(x_{i_1}, x_{i_2}, \dots, x_{i_k}), i \in I$$

be a system of equations in the variables  $\{x_j : j \in V\}$  with the following properties:

- (1) For each  $j \in V$ , the variable  $x_j$ , has a finite domain,  $D_j$ .
- (2) Given any finite number of variables, there is an equation in the system (\*) which contains those variable and possibly others.
- (3) For each equation in the system (\*) there is an equation in (\*) with the same, or possibly more, variables which has a solution.

Then, the system (\*) has a partial solution. (The family of equations  $(x_j = d_j)_{j \in V}$  with  $d_j \in D_j$  is called a *partial solution* of the system (\*) if for every finite subfamily of equations

$$(**) \quad x_i = d_i, \dots, x_k = d_k,$$

there is an equation in the system (\*) with the variables  $x_i, \dots, x_k$ , and possibly other variables, such that (\*\*) is part of the complete solution of that equation.) Abian [1973] and note 149.

[14 CZ] Wallman's Lemma. Assume  $\mathcal{C} = \{C_j : j \in J\}$  is a family of finite sets. For each  $j \in J$ , let  $b_j = \bigcup C_j$  and assume that the family  $\{b_j : j \in J\}$  has the finite intersection property. Then there is a choice function  $f$  for  $\mathcal{C}$  such that  $\{f(C_j) : j \in J\}$  has the finite intersection property. Cowen [1983].

[43 AD] Every Frechet complete (pseudo)metric space is Baire. Herrlich/Keremedis [1999a], notes 10 and 28.

[43 AE] Countable products of compact Hausdorff spaces are Baire. Herrlich/Keremedis [1999a] and note 28.

[43 AF] Every infinite branching poset (a partially ordered set in which each element has at least two lower bounds) has either a countably infinite chain or a countably infinite antichain. Keremedis [1999a].

[67 W] Every closed (open) filter in a  $T_1$  topological space has a well orderable filter base. Keremedis/Tachtsis [1999c] and note 10.

[67 X] Every open ultrafilter in  $T_1$  topological space has a well orderable filter base. Keremedis/Tachtsis [1999c] and note 10.

[67 Y] For every set  $A \neq \emptyset$ , every filter contained in  $\mathcal{P}(A)$  has a well orderable filter base. Herrlich/Tachtsis [1999a] and note 10.

[67 Z] Every closed (open) filter in a dense in itself  $T_1$  topological space has a well orderable filter base. Keremedis/Tachtsis [1999a] and note 10.

[67 AA] If  $(X, T)$  is a  $T_2$  topological space and  $\mathcal{B}$  is a lattice of closed sets, then every maximal  $\mathcal{B}$  filter has a well orderable filter base. Keremedis/Tachtsis [1999a] and note 10.

[82 C]  $P\text{-}\aleph_0$ : For every infinite set  $X$ , there is a partition of  $X$  of cardinality  $\aleph_0$ . González [1995a].

[126 L] No maximal closed free filter in a  $T_1$  topological space has a countable filter base. Keremedis/Tachtsis [1999c] and note 10.

[126 M] Weierstrass compact pseudometric spaces are countably compact. Keremedis [1999c] and notes 6 and 10.

[126 N] Weierstrass compact pseudometric spaces are compact. Keremedis [1999c] and notes 6 and 10.

[126 O] Every compact pseudometric space has a dense subset which can be written as a countable union of finite sets. Keremeids [1999c] and note 10.

[126 P] Every compact pseudometric space has a dense subset which can be written as a well ordered union of finite sets. Keremeids [1999c] and note 10.

**FORM 360.** A system of linear equations over the field  $\{0, 1\}$  has a solution, if and only if every finite subsystem has a solution. Brunner [1999].

[374 B( $n+1$ )] Countable products of Hausdorff spaces  $X_m$ , with  $|X_m| \leq n+1$ ,  $m \in \omega$ , are compact. Herrlich/Keremedis [1999b].

[374  $C(n+1)$ ] Countable products of Hausdorff spaces  $X_m$ , with  $|X_m| \leq n+1$ ,  $m \in \omega$ , are Baire. Herrlich/Keremedis [1999b] and note 28.

**FORM 384.** Closed Filter Extendability for  $T_1$  Spaces. Every closed filter in a  $T_1$  topological space can be extended to a maximal closed filter. Herrlich/Steprans [1997] and note 10.

**FORM 385.** Countable Ultrafilter Theorem: Every proper filter with a countable base over a set  $S$  (in  $\mathcal{P}(S)$ ) can be extended to an ultrafilter. Herrlich/Keremedis [1999a].

**FORM 386.** Every B compact (pseudo)metric space is Baire. Herrlich/Keremedis [1999a], notes 6, 10, and 28.

**FORM 387.** DPO: Every infinite set has a non-trivial, dense partial order. (A partial ordering  $<$  on a set  $X$  is dense if  $(\forall x, y \in X)(x < y \rightarrow (\exists z \in X)(x < z < y))$  and is non-trivial if  $(\exists x, y \in X)(x < y)$ ). González [1995a].

**FORM 388.** Every infinite branching poset (a partially ordered set in which each element has at least two lower bounds) has either an infinite chain or an infinite antichain. Keremedis [1999a].

**FORM 389.**  $C(\aleph_0, 2, \mathcal{P}(\mathbb{R}))$ : Every denumerable family of two element subsets of  $\mathcal{P}(\mathbb{R})$  has a choice function. Keremedis [1999b].

**FORM 390.** Every infinite set can be partitioned either into two infinite sets or infinitely many sets, each of which has at least two elements. Ash [1983] and Howard/Yorke [1989].

**FORM 391.**  $C(\infty, LO)$ : Every set of non-empty linearly orderable sets has a choice function.

**FORM 392.**  $C(LO, LO)$ : Every linearly ordered set of linearly orderable sets has a choice function.

**FORM 393.**  $C(LO, WO)$ : Every linearly ordered set of non-empty well orderable sets has a choice function.

**FORM 394.**  $C(WO, LO)$ : Every well ordered set of non-empty linearly orderable sets has a choice function.

**FORM 395.**  $MC(LO, LO)$ : For each linearly ordered family of non-empty linearly orderable sets  $X$ , there is a function  $f$  such that for all  $x \in X$   $f(x)$  is non-empty, finite subset of  $x$ .

**FORM 396.**  $MC(LO, WO)$ : For each linearly ordered family of non-empty well orderable sets  $X$ , there is a function  $f$  such that for all  $x \in X$   $f(x)$  is non-empty, finite subset of  $x$ .

**FORM 397.**  $MC(WO, LO)$ : For each well ordered family of non-empty linearly orderable sets  $X$ , there is a function  $f$  such that for all  $x \in X$   $f(x)$  is non-empty, finite subset of  $x$ .

**FORM 398.**  $KW(LO, \infty)$ , The Kinna-Wagner Selection Principle for a linearly ordered family of sets: For every linearly ordered set  $M$  there is a function  $f$  such that for all  $A \in M$ , if  $|A| > 1$  then  $\emptyset \neq f(A) \subsetneq A$ .

**FORM 399.**  $KW(\infty, LO)$ , The Kinna-Wagner Selection Principle for a set of linearly orderable sets: For every set of linearly orderable sets  $M$  there is a function  $f$  such that for all  $A \in M$ , if  $|A| > 1$  then  $\emptyset \neq f(A) \subsetneq A$ .

**FORM 400.**  $KW(LO, LO)$ , The Kinna-Wagner Selection Principle for a linearly ordered set of linearly orderable sets: For every linearly ordered set of linearly orderable sets  $M$  there is a function  $f$  such that for all  $A \in M$ , if  $|A| > 1$  then  $\emptyset \neq f(A) \subsetneq A$ .

**FORM 401.**  $KW(LO, < \aleph_0)$ , The Kinna-Wagner Selection Principle for a linearly ordered set of finite sets: For every linearly ordered set of finite sets  $M$  there is a function  $f$  such that for all  $A \in M$ , if  $|A| > 1$  then  $\emptyset \neq f(A) \subsetneq A$ .

**FORM 402.**  $KW(WO, LO)$ , The Kinna-Wagner Selection Principle for a well ordered set of linearly orderable sets: For every well ordered set of linearly orderable sets  $M$  there is a function  $f$  such that for all  $A \in M$ , if  $|A| > 1$  then  $\emptyset \neq f(A) \subsetneq A$ .

**FORM 403.**  $KW(LO, WO)$ , The Kinna-Wagner Selection Principle for a linearly ordered set of well orderable sets: For every linearly ordered set of well orderable sets  $M$  there is a function  $f$  such that for all  $A \in M$ , if  $|A| > 1$  then  $\emptyset \neq f(A) \subsetneq A$ .

**FORM 404.** Every infinite set can be partitioned into infinitely many sets, each of which has at least two elements. Ash [1983] and Howard/Yorke [1989].

**FORM 405.** Every infinite set can be partitioned into sets each of which is countable and has at least two elements.