

Additions to Part I: Numerical List of Forms

[0 AR] For all cardinals m and n , if $2m = 2n$, then $m = n$. Sierpiński [1922]. (Brought to our attention by W. Felscher.)

[0 AS] The Modified Ascoli Theorem. For any set F of continuous functions from \mathbb{R} to \mathbb{R} , the following conditions are equivalent:

- (1) Each sequence in F has a subsequence that converges continuously to some continuous function (not necessarily in F).
- (2) (a) For each countable subset G of F and each $x \in \mathbb{R}$, the set $G(x) = \{g(x) : g \in G\}$ is bounded, and
(b) Each countable subset of F is equicontinuous.

Rhineghost [2000] and note 10

Add a reference to Keremedis in form [1 DG].

[1 DG] Vector Space Kinna-Wagner Principle: For every family $V = \{V_i : i \in K\}$ of non-trivial vector spaces there is a family $F = \{F_i : i \in K\}$ such that for each $i \in K$, F_i is a non-empty, independent subset of V_i . Keremedis [1999e] and note 127.

[14 DA] Keimel's Representation Theorem. Any hyperarchimedean l -group can be imbedded into a Boolean product of a family of simple, abelian l -groups. Gluschankof [1995] and note 151.

[14 DB] If G is an achimedean l -group with a weak unit, then G can be imbedded into $D(X)$ where X is a locally compact topological space and $D(X)$ is the set of continuous maps $f : X \rightarrow \mathbb{R} \cup \{-\infty, \infty\}$ such that the inverse image of \mathbb{R} is dense in X . Gluschankof [1995] and note 151.

[14 DC] Clifford's Theorem. An abelian l -group is isomorphic to a subdirect product of totally ordered abelian groups. Gluschankof [1995] and note 151.

[14 DD] Lorenzen's Theorem. A representable l -group is isomorphic to a subdirect product of totally ordered groups. Gluschankof [1995] and note 151.

[14 DE] Holland's Theorem. Any l -group is isomorphic to a subdirect product of transitive l -groups. Gluschankof [1995] and note 151.

[14 DF] Any l -group can be imbedded into the l -preserving permutations of a totally ordered set. Gluschankof [1995] and note 151.

[14 DG] In any hyperarchimedean l -group any proper l -ideal can be extended to a prime one. Gluschankof [1989] and note 151.

[43 AI] Every countably well-founded relation is well-founded. (A relation R is (*countably*) *well-founded* if every (countable) set in the domain of R has an R -minimal element.) Diener [1994].

[94 Q] The Classical Ascoli Theorem. For any set F of continuous functions from \mathbb{R} to \mathbb{R} , the following conditions are equivalent:

- (1) Each sequence in F has a subsequence that converges continuously to some continuous function (not necessarily in F).
- (2) (a) For each $x \in \mathbb{R}$, the set $F(x) = \{f(x) : f \in F\}$ is bounded, and
(b) F is equicontinuous.

Rhineghost [2000] and note 10

[94 R] Weak Determinateness. If A is a subset of $\mathbb{N}^{\mathbb{N}}$ with the property that

$$(\forall a \in A)(\forall x \in \mathbb{N}^{\mathbb{N}}) (x_n = a_n \text{ for } n = 0 \text{ and } n \text{ odd} \rightarrow x \in A)$$

Then in the game $G(A)$ one of the two players has a winning strategy. Rhineghost [2000] and note 153. **FORM 408.** If $\{f_i : i \in I\}$ is a family of functions such that for each $i \in I$, $f_i \subseteq E \times W$, where E and W are non-empty sets, and \mathcal{B} is a filter base on I such that

1. For all $B \in \mathcal{B}$ and all finite $F \subseteq E$ there is an $i \in I$ such that f_i is defined on F , and
2. For all $B \in \mathcal{B}$ and all finite $F \subseteq E$ there exist at most finitely many functions on F which are restrictions of the functions f_i with $i \in I$, then there is a function f with domain E such that for each finite $F \subseteq E$ and each $B \in \mathcal{B}$ there is an $i \in I$ such that $f|F = f_i|F$. Felscher [1964].

FORM 409. Suppose (G, Γ) is a locally finite graph (i.e. G is a non-empty set and Γ is a function from G to $\mathcal{P}(G)$ such that for each $x \in G$, $\Gamma(x)$ and $\Gamma^{-1}\{x\}$ are finite), K is a finite set of integers, and T is a function mapping subsets of K into subsets of K . If for each finite subgraph (A, Γ_A) there is a function ϕ_A such that for each $x \in A$, $\phi_A(x) \in T(\phi_A[\Gamma_A(x)])$, then there is a function ϕ such that for all $x \in G$, $\phi(x) \in T(\phi[\Gamma(x)])$. Foster [1964].

FORM 410. RC (Reflexive Compactness): The closed unit ball of a reflexive normed space is compact for the weak topology. Delhommé/Morillon [2000] and note 23.

FORM 411. RCuc (Reflexive Compactness for uniformly convex Banach spaces): The closed unit ball of a uniformly convex Banach space is compact for the weak topology. Delhommé/Morillon [2000] and note 23.

FORM 412. RCh (Reflexive Compactness for Hilbert spaces): The closed unit ball of a Hilbert space is compact for the weak topology. Delhommé/Morillon [2000] and note 23.