Additions to Part I: Numerical List of Forms

[0 AQ] If $E$ is a separable normed topological vector space, then for every continuous sublinear functional $p$ on $E$ there is a linear functional $f$ on $E$ such that $f \leq p$. Dodu/Morillon [1999].


[1 DA] Every large subspace of the spectrum of a compact frame is compact. (A subspace is large if it contains all minimal elements.) Banaschewski [1990] and note 29.

[1 DB] Any large homomorphic image of a compact frame is compact. (A frame homomorphism $\phi : L \to M$ is called large if $\phi(s) < e$ (the unit) for all maximal elements $s \in L$.) Banaschewski [1990] and note 29.

[1 DC] In any complete lattice with at least two elements there exists an ultrafilter. Banaschewski [1961].

[1 DD] If $(L_i)_{i \in I}$ is a family of complete lattices, each of which has at least two elements, then there exists a family $(M_i)_{i \in I}$ of filters such that $M_i$ is an ultrafilter in $L_i$. Banaschewski [1961].

[1 DE] Let $L$ be a complete distributive lattice with at least two elements and let $A$ be a proper subset of $L$ such that for all $x \in L$ and $a \in A$, $x \leq a$ implies $x \in a$. Then there is an ultrafilter $M$ disjoint from $A$. Banaschewski [1961].

[1 DF] For any set $X$, if $C$ is a set of conditionally $\cap$-closed subsets of $\mathcal{P}(X)$, then $C$ contains a maximal filter (not necessarily proper). (If $C \subseteq \mathcal{P}(X)$, $C$ is called conditionally $\cap$-closed if for all $A, B, C$ in $C$, $C \subseteq A \cap B$ implies $A \cap B \in C$.) Banaschewski [1961].

(Keremedis [1999d] has shown that form 346 is equivalent to AC. Form [1 DG] is the old 346.)

[1 DG] Vector Space Kinna-Wagner Principle: For every family $V = \{V_i : i \in K\}$ of non-trivial vector spaces there is a family $F = \{F_i : i \in K\}$ such that for each $i \in K$, $F_i$ is a non-empty, independent subset of $V_i$. Note 127.

[43 AG] Let $B$ be a Boolean algebra, $b$ a non-zero element of $B$ and $\{A_i : i \in \omega\}$ a sequence of subsets of $B$ such that for each $i \in \omega$, $A_i$ has a supremum $a_i$. Then there exists a filter $D$ in $B$ such that $b \in D$ and, for each $i \in \omega$, if $a_i \in D$, then $D \cap A_i \neq \emptyset$. Bacsich [1972b].

[43 AH] Ekeland’s Variational Principle: If $(E, d)$ is a non-empty complete metric space, $f : E \to \mathbb{R}$ is lower semi-continuous and bounded from below, and $\epsilon$ is a positive real number, then there exists $a \in E$ such that for all $x \in E$, $f(a) \leq f(x) + \epsilon d(x, a)$. Dodu/Morillon [1999] and note 28.

[52 L] If $E$ is a topological vector space then for every sublinear functional $p$ on $E$ there is a linear functional $f$ on $E$ such that $f \leq p$. Fossey/Morillon [1998].

[52 M] If $E$ is a topological vector space, $p$ is a continuous sublinear functional on $E$, and $S$ is a subspace of $E$ such that $f$ is a linear functional on $S$ with $f \leq p$, then $f$ can be extended to $f^* : E \to \mathbb{R}$ such that $f^*$ is linear and $f^* \leq p$. Fossey/Morillon [1998].
[52 N] If $E$ is a topological vector space, $C$ is a non-empty open convex subset of $E$ such that $C \cap O = \emptyset$, then there exists a linear functional $f$ on $E$ such that for all $x \in O$, $f(x) < \inf_{z \in C} f(z)$. Dodu/Morillon [1999].

[52 O] If $E$ is a topological vector space, $C$ is a non-empty convex subset of $E$, and $O$ is a non-empty open convex subset of $E$ such that $C \cap O = \emptyset$, then there exists a linear functional $f$ on $E$ such that for all $x \in O$, $f(x) < \inf_{z \in C} f(z)$. Dodu/Morillon [1999].

[52 P] If $E$ is a topological vector space, $a \in E$, and $O$ is a non-empty open convex subset of $E$ such that $a \notin O$, then there exists a linear functional $f$ on $E$ such that for all $x \in O$, $f(x) < f(a)$. Dodu/Morillon [1999].

[52 Q] If $E$ is a topological vector space, $C$ and $O$ two non-empty disjoint convex subsets of $E$ and $O$ is open, then there exists a linear functional $f$ on $E$ such that $f[O] < f[C]$. Dodu/Morillon [1999].

[52 R] If $E$ is a topological vector space, $C$ and $O$ two non-empty disjoint open convex subsets of $E$, then there exists a linear functional $f$ on $E$ such that $f[O] < f[C]$. Dodu/Morillon [1999].

[52 S] If $E$ is a topological vector space, $C$ is a non-empty closed convex subset of $E$, and $K$ is a non-empty compact convex subset of $E$, then there exists a linear functional $f$ on $E$ such that $\sup_{x \in K} f(x) < \inf_{z \in C} f(z)$. Dodu/Morillon [1999].

Replace the old form 346 by the following:

**FORM 346.** If $V$ is a vector space without a finite basis then $V$ contains an infinite, well ordered, linearly independent subset. Keremedis [1999d].

**FORM 406.** The product of compact Hausdorff spaces is countably compact. Alas [1994].

**FORM 407.** Let $B$ be a Boolean algebra, $b$ a non-zero element of $B$ and $\{A_i : i \in \omega\}$ a sequence of subsets of $B$ such that for each $i \in \omega$, $A_i$ has a supremum $a_i$. Then there exists an ultrafilter $D$ in $B$ such that $b \in D$ and, for each $i \in \omega$, if $a_i \in D$, then $D \cap A_i \neq \emptyset$. Bacsich [1972b].