Additions to Part I: Numerical List of Forms

[**0 AQ**] If E is a separable normed topological vector space, then for every continuous sublinear functional p on E there is a linear functional f on E such that $f \leq p$. Dodu/Morillon [1999].

[**1 CZ**] Every compact frame has a maximal element. Banaschewski [1990] and note 29.

[**1 DA**] Every large subspace of the spectrum of a compact frame is compact. (A subspace is *large* if it contains all minimal elements.) Banaschewski [1990] and note 29.

[**1 DB**] Any large homomorphic image of a compact frame is compact. (A frame homomorphism $\phi : L \to M$ is called *large* if $\phi(s) < e$ (the unit) for all maximal elements $s \in L$.) Banaschewski [1990] and note 29.

[**1 DC**] In any complete lattice with at least two elements there exists an ultrafilter. Banaschewski [1961].

[**1 DD**] If $(L_i)_{i\in I}$ is a family of complete lattices, each of which has at least two elements, then there exists a family $(M_i)_{i\in I}$ of filters such that M_i is an ultrafilter in L_i. Banaschewski [1961].

[**1 DE**] Let L be a complete distributive lattice with at least two elements and let A be a proper subset of L such that for all $x \in L$ and $a \in A$, $x \leq a$ implies $x \in a$. Then there is an ultrafilter M disjoint form A. Banaschewski [1961].

[**1 DF**] For any set X, if $\mathcal{C}$ is a set of conditionally $\cap$-closed subsets of $\mathcal{P}(X)$, then $\mathcal{C}$ contains a maximal filter (not necessarily proper). (If $\mathcal{C} \subseteq \mathcal{P}(X)$, $\mathcal{C}$ is called *conditionally* $\cap$*-closed* if for all A, B, C in $\mathcal{C}$, $C \subseteq A \cap B$ implies $A \cap B \in \mathcal{C}$.) Banaschewski [1961].

(Keremedis [1999d] has shown that form 346 is equivalent to AC. Form [1 DG] is the old 346.)

[**1 DG**] Vector Space Kinna-Wagner Principle: For every family $V = \{V_i : i \in K\}$ of non-trivial vector spaces there is a family $F = \{F_i : i \in K\}$ such that for each $i \in K$, F_i is a non-empty, independent subset of V_i. Note 127.

[**43 AG**] Let B be a Boolean algebra, b a non-zero element of B and $\{A_i : i \in \omega\}$ a sequence of subsets of B such that for each $i \in \omega$, A_i has a supremum a_i. Then there exists a filter D in B such that $b \in D$ and, for each $i \in \omega$, if $a_i \in D$, then $D \cap\ A_i \neq \emptyset$. Bacsich [1972b].

[**43 AH**] Ekeland's Variational Principle: If (E, d) is a non-empty complete metric space, $f : E \to \mathbb{R}$ is lower semi-continuous and bounded from below, and ϵ is a positive real number, then there exists $a \in E$ such that for all $x \in E$, $f(a) \leq f(x) + \epsilon d(x, a)$. Dodu/Morillon [1999] and note 28.

[**52 L**] If E is a topological vector space then for every sublinear functional p on E there is a linear functional f on E such that $f \leq p$. Fossy/Morillon [1998].

[**52 M**] If E is a topological vector space, p is a continuous sublinear functional on E, and S is a subspace of E such that f is a linear functional on S with $f \leq p$, then f can be extended to $f^* : E \to \mathbb{R}$ such that f^* is linear and $f^* \leq p$. Fossy/Morillon [1998].

[**52 N**] If E is a topological vector space, C is a an affine subspace of E, and O is a non-empty open convex subset of E such that $C \cap O = \emptyset$, then there exists a linear functional f on E such that for all $x \in O$, $f(x) < \inf_{z \in C} f(z)$. Dodu/Morillon [1999].

[**52 O**] If E is a topological vector space, C is a non-empty convex subset of E, and O is a non-empty open convex subset of E such that $C \cap O = \emptyset$, then there exists a linear functional f on E such that for all $x \in O$, $f(x) < \inf_{z \in C} f(z)$. Dodu/Morillon [1999].

[**52 P**] If E is a topological vector space, $a \in E$, and O is a non-empty open convex subset of E such that $a \notin O$, then there exists a linear functional f on E such that for all $x \in O$, $f(x) < f(a)$. Dodu/Morillon [1999].

[**52 Q**] If E is a topological vector space, C and and O two non-empty disjoint convex subsets of E and O is open, then there exists a linear functional f on E such that $f[O] < f[C]$. Dodu/Morillon [1999].

[**52 R**] If E is a topological vector space, C and and O two non-empty disjoint open convex subsets of E, then there exists a linear functional f on E such that $f[O] < f[C]$. Dodu/Morillon [1999].

[**52 S**] If E is a topological vector space, C is a non-empty closed convex subset of E, and K is a non-empty compact convex subset of E, then there exists a linear functional f on E such that $\sup_{x \in K} f(x) < \inf_{z \in C} f(z)$. Dodu/Morillon [1999].

Replace the old form 346 by the following:

FORM 346. If V is a vector space without a finite basis then V contains an infinite, well ordered, linearly independent subset. Keremedis [1999d].

FORM 406. The product of compact Hausdorff spaces is countably compact. Alas [1994].

FORM 407. Let B be a Boolean algebra, b a non-zero element of B and $\{A_i : i \in \omega\}$ a sequence of subsets of B such that for each $i \in \omega$, A_i has a supremum a_i. Then there exists an ultrafilter D in B such that $b \in D$ and, for each $i \in \omega$, if $a_i \in D$, then $D \cap \ A_i \neq \emptyset$. Bacsich [1972b].