Additions to Part II: Topical List of Forms

1. Add a new category:

PARTITIONS

4. Every infinite set is the union of some disjoint family of denumerable subsets. (Denumerable means $\cong \aleph_0$.) Whitehead [1902].

[**14 AC**] (Depends on k, $n \in \omega$ with $k \geq 2$, $n \geq 2$ and $n + k \geq 5$.) $P(k, n)$: If X is a set and P is a property of subsets of X of n character (that is, $\forall y \subseteq X(P(y)$ iff $\forall z \subseteq y(|z| \leq n \rightarrow P(z))))$, then if every finite subset of X can be partitioned into k or fewer P-sets (that is, sets z such that $P(z)$) then X can be partitioned into k or fewer P-sets. Cowen [1982].

[**14 AD**] (Depends on $k \in \omega$, $k \geq 2$) $P(k)$: If X is a set and P is a property of subsets of X of finite character (that is, $\forall y \subseteq X(P(y)$ iff $\forall z \subseteq y(z$ finite $\rightarrow P(z))))$, then if every finite subset of X can be partitioned into k or fewer P-sets (that is, sets z such that $P(z)$), then X can be partitioned into k or fewer P-sets. Cowen [1982].

17. Ramsey's Theorem I: If A is an infinite set and the family of all 2 element subsets of A is partitioned into 2 sets X and Y, then there is an infinite subset $B \subseteq A$ such that all 2 element subsets of B belong to X or all 2 element subsets of B belong to Y. (Also, see form 325.) Jech [1973b] p 164 prob 11.20 and Ramsey [1929].

[**40 B**] For all ordinals α, the $\aleph_\alpha$ partition principle holds: For every ordinal α and every cardinal κ, if $\aleph_\alpha \leq^* \kappa$, then $\aleph_\alpha \leq \kappa$. Pelc [1978], Banaschewski/Moore [1990], and note 69.

64. $E(I, Ia)$ (Howard/Yorke [1989]): There are no amorphous sets. (Equivalently, every infinite set is the union of two disjoint infinite sets.) Levy [1958], notes 57 and 94.

75. If a set has at least two elements, then it can be partitioned into well ordered subsets, each of which has at least two elements. Higasikawa [1995].

[**82 C**] P-$\aleph_0$: For every infinite set X, there is a partition of X of cardinality $\aleph_0$. González [1995a].

[**88 A**] $P(2, 2)$: If X is a set and P is a property of subsets of X of 2 character (that is, $\forall y \subseteq X(P(y)$ iff $\forall z \subseteq y(|z| \leq 2 \rightarrow P(z))))$ then if every finite subset of X can be partitioned into two or fewer P-sets (that is, sets z such that $P(z)$) then X can be partitioned into two or fewer P-sets. Cowen [1982].

100. Weak Partition Principle: For all sets x and y, if $x \precsim^* y$, then it is not the case that $y \prec x$. Lindenbaum/Tarski [1926] and note 69.

101. Partition Principle: If S is a partition of M, then $S \precsim M$. Sierpiński [1947].

129. For every infinite set A, A admits a partition into sets of order type $\omega^* + \omega$. (For every infinite A, there is a set $\{\langle C_j, <_j\rangle : j \in J\}$ such that $\{C_j : j \in J\}$ is a partition of A and for each $j \in J$, $<_j$ is an ordering of C_j of type $\omega^* + \omega$.) Brunner [1984f].

137(k). Suppose $k \in \omega - \{0\}$. If f is a 1-1 map from $k \times X$ into $k \times Y$ then there are partitions $X = \bigcup_{i<k} X_i$ and $Y = \bigcup_{i<k} Y_i$ of X and Y such that f maps $\bigcup_{i<k}(\{i\} \times X_i)$ onto $\bigcup_{i<k}(\{i\} \times Y_i)$. Truss [1984].

[**137 A(k)**] For all X and $Y \subseteq \mathbb{R}$ form 137(k) holds. Truss [1984]

138(k). Suppose $k \in \omega$. If f is a partial map from $k \times Y$ onto $k \times X$ (that is, the domain is a subset of $k \times Y$), then there are partitions $X = \bigcup_{i<k} X_i$ and $Y = \bigcup_{i<k} Y_i$ of X and Y such that f maps $\bigcup_{i<k}(\{i\} \times Y_i)$ onto $\bigcup_{i<k}(\{i\} \times X_i)$. Truss [1984].

[**138 A(k)**] For all standard X and Y, form 138(k) holds. Truss [1984]. This is equivalent to 138(k) in ZF^0.

[**140 A**] (Form 137 with $k = 2$): If f is a 1-1 map from $2 \times X$ into $2 \times Y$ then there are partitions $X = X_0 \cup X_1$ and $Y = Y_0 \cup Y_1$ of X and Y such that f maps $(\{0\} \times X_0) \cup (\{1\} \times X_1)$ onto $(\{0\} \times Y_0) \cup (\{1\} \times Y_1)$. Truss [1984].

152. $D_{\aleph_0}$: Every non-well-orderable set is the union of a pairwise disjoint, well orderable family of denumerable sets. Brunner/Howard [1992]. (See note 27 for D_κ, κ a well ordered cardinal.)

203. C(disjoint,$\subseteq \mathbb{R}$): Every partition of $\mathcal{P}(\omega)$ into non-empty subsets has a choice function. Truss [1978].

209. There is an ordinal α such that for all X, if X is a denumerable union of denumerable sets then $\mathcal{P}(X)$ cannot be partitioned into $\aleph_\alpha$ non-empty sets. Morris [1970].

224. There is a partition of the real line into $\aleph_1$ Borel sets $\{B_\alpha : \alpha < \aleph_1\}$ such that for some $\beta < \aleph_1$, $\forall \alpha < \aleph_1$, $B_\alpha \in G_\beta$. (G_β for $\beta < \aleph_1$ is defined by induction, $G_0 = \{A : A$ is an open subset of $\mathbb{R}\}$ and for $\beta > 0$,
$G_\beta = \{\bigcup_{i=0}^{\infty} A_i : (\forall i \in \omega)(\exists \xi < \beta)(A_i \in G_\xi)\}$ if β is even and
$G_\beta = \{\bigcap_{i=0}^{\infty} A_i : (\forall i \in \omega)(\exists \xi < \beta)(A_i \in G_\xi)\}$ if β is odd.)
Stern [1979].

282. $\omega \not\to (\omega)^\omega$. Erdös/Rado [1952], Kleinberg/Seiferas [1973] and note 97.

296. Part-∞: Every infinite set is the disjoint union of infinitely many infinite sets. Gonzalez [1995b], Pincus [1997].

309. The Banach-Tarski Paradox: There are three finite partitions $\{P_1, \ldots, P_n\}$, $\{Q_1, \ldots, Q_r\}$ and $\{S_1, \ldots, S_n, T_1, \ldots, T_r\}$ of $B^3 = \{x \in \mathbb{R}^3 : |x| \leq 1\}$ such that P_i is congruent to S_i for $1 \leq i \leq n$ and Q_i is congruent to T_i for $1 \leq i \leq r$. Banach/Tarski [1924].

325. Ramsey's Theorem II: $\forall n, m \in \omega$, if A is an infinite set and the family of all m element subsets of A is partitioned into n sets $S_j, 1 \leq j \leq n$, then there is an infinite subset $B \subseteq A$ such that all m element subsets of B belong to the same S_j. (Also, see form 17.) Ramsey [1929].

347. Idemmultiple Partition Principle: If y is idemmultiple ($2 \times y \approx y$) and $x \precsim^* y$, then $x \precsim y$. Higasikawa [1995] and note 69.

369. If $\mathbb{R}$ is partitioned into two sets, at least one of them has cardinality $2^{\aleph_0}$. Luzin/Sierpiński [1917].

390. Every infinite set can be partitioned either into two infinite sets or infinitely many sets, each of which has at least two elements. Ash [1983] and Howard/Yorke [1989].

404. Every infinite set can be partitioned into infinitely many sets, each of which has at least two elements. Ash [1983] and Howard/Yorke [1989].

405. Every infinite set can be partitioned into sets each of which is countable and has at least two elements.