Additions to Part III: Models

I. In $\mathcal{M}40(\kappa)$ replace the part following:

Since 43 + 14 is equivalent to 345 ([345 A]), form 345 is true.

with

Since 8 is true and 384 + 8 implies 1, form 384 is false.

$\mathcal{M}40(\kappa) \models$ 14, 30, 43, 60, 87(α) for $\aleph_\alpha < \kappa$, 165, 295, and 345, but 51, 91, 144, 152, 163, 253, 286, and 384 are false. References Pincus [1977a], [1977b], [1997], Brunner [1982a], Herrlich/Steprans [1997], Morris [1969], notes 18, 120(26, 28, 34, 43, 45, and 59) and 121.

II. A new model

$\mathcal{N}56$: Howard's model III: Assume the the atoms are indexed as follows: $A = \{a(i,j) : i \in \mathbb{Q}$ and $j \in \omega\}$. For each $i \in \mathbb{Q}$, let $A_i = \{a(i,j) : j \in \omega\}$. (Call A_i the ith block.) Let

$$\mathcal{G} = \{\phi : A \to A : (\forall i \in \mathbb{Q})(\exists i' \in \mathbb{Q})(\phi(A_i) = A_{i'}) \text{ and } \{i \in \mathbb{Q} : (\exists j)(\phi(a(i,j)) \neq a(i,j))\} \text{ is bounded}\}.$$

Here bounded means bounded in the usual ordering on $\mathbb{Q}$. ($\mathcal{G}$ is the set of all permutations of ϕ of A such that ϕ of any block is a block and the set of blocks on which ϕ is not the identity is bounded.) S is the set of subsets of A of the form $\bigcup_{i \in E} A_i$ where E is a bounded subset of $\mathbb{Q}$. Form 40 ($C(WO, \infty)$) is true using a proof similar to the proof of 40 in $\mathcal{N}33$ given in Howard/Rubin/Rubin [1973]. By note 104, form 3 ($2m = m$) is false. Form 62 ($C(\infty, < \aleph_0)$) is false since the family of finite, non-empty subsets of A has no choice function. Form 67 (MC) is false since the set of blocks has no multiple choice function.

$\mathcal{N}56 \models$ 6, 37, 40, 91, 92, 130, 182, 189, 190, 191, 273, 305, 309, 313, 361, 363, 368, and 369, but 3, 15, 62, 67, 152, and 192 are false. References Howard [1990], Howard/Rubin/Rubin [1973], and notes 18, 120(46).

III. Add to toc_mod.tex

55. $\mathcal{N}55$: Keremedis/Tachtsis Model.

(6, 9, 37, 91, 92, 130, 182, 189, 190, 191, 273, 305, 309, 313, 361, 363, 368, and 369 are true but, 15, 116, 131, 154, 165, and 343 are false.)

56. $\mathcal{N}56$: Howard's Model III.

(6, 37, 40, 91, 92, 130, 182, 189, 190, 191, 273, 305, 309, 313, 361, 363, 368, and 369, but 3, 15, 62, 67, 152, and 192 are false.)

IV. In $\mathcal{M}2$, in the beginning of the description change "the Boolean Prime Ideal Theorem (14) is false." to "the countable ultrafilter theorem (385) is false."

At the end of $\mathcal{M}2$, add 385 to the list of theorems that are false.

VI. In $\mathcal{M}1$, right after "so 277 is true." insert the sentence:

Since $\mathbb{R}$ cannot be well ordered in this model and form 277 is true, it follows that 369 ($\mathbb{R}$ is not decomposable) is false.

Also, add 369 to the list of froms that are false.

VI. Change the last part of N2 to:

Keremedis [1999] has shown that 216 (Every infinite tree has either an infinite chain or an infinite antichain.) and 388 (Every infinite branching poset (a partially ordered set in which each element has at least two lower bounds) has either an infinite chain or an infinite antichain.) are both false in $\mathcal{N}2$.

$\mathcal{N}2 \models$ 6, 37, 65, 67, 83, 99, 130, 163, 191, 273, 305, 313, 334, 361, and 363, but 18, 45(n), 80, 98, 128, 154, 163, 164, 198, 216, 344, 358, and 388 are false. References Brunner [1982a], [1983c], [1984b], [1984f], Fraenkel [1922], Hickman [1978b], Howard [1984b], Jech [1973b], Keremedis [1999], Pincus [1972a],[1972b], notes 18, 93, 95, 105, and 120(2 and 56).

V. Change the first part of the description of $\mathcal{M}7$ to the following:

$\mathcal{M}7$: Cohen's Second Model. There are two denumerable subsets $U = \{U_i : i \in \omega\}$ and $V = \{V_i : i \in \omega\}$ of $\mathcal{P}(\mathbb{R})$ (neither of which is in the model) such that for each $i \in \omega$, U_i and V_i cannot be distinguished in the model. Therefore, in this model there is a denumerable set of pairs of subsets of $\mathbb{R}$ whose union has no denumerable subset, so form 18 is false, and $C(\aleph_0, 2, \mathcal{P}(\mathbb{R}))$ (389) is also false.

Also in $\mathcal{M}7$, delete 358 from the list of forms that are false and add 389 to this list.

VI. Add as the last part of the descriptions of $\mathcal{M}6$:

Sageev proves that alephs are preserved in this model so, since form 170 ($\aleph_1 \leq 2^{\aleph_0}$ is true in the ground model, it is true in $\mathcal{M}6$. It also follows that 34 ($\aleph_1$ is regular) is true. (See Howard/Keremedis/Rubin/Stanley/Tachtsis [1999] Lemma 5.)

Also, add 34 and 170 to the list of forms that are true in this model and add Howard/Keremedis/Rubin/ [1999] to the list of references.

VII. In $\mathcal{M}12(\aleph)$ after "(169 is false).", insert the following:

It follows from [0 AB] that every perfect subset of $\mathbb{R}$ has cardinality $2^{\aleph_0}$. Thus, since 169 is false, form 369 (If $\mathbb{R}$ is partitioned into two sets, at least one of them has cardinality $2^{\aleph_0}$.) is true.

Add 369 to the list of forms that are true in this model.

VIII. At the end of $CalN1$, add:

Form 390 is false because the set of atoms can neither be partitioned into two infinite sets nor can it be partitioned into infinitely many sets, each of which has at least two elements.

Add form 390 to the list of forms that is false in $\mathcal{N}1$.