Additions to Part IV: Notes

Add the following as a new paragraph at the end of note 40:

The following are definitions for [94 S]. Suppose $x \in \mathbb{R}$ and $X \subseteq \mathbb{R}$.

(a) x is called an *accumulation point* of X if every open neighborhood of x contains a point in $X - \{x\}$.

(b) x is called a *cluster point* of X if every open neighborhood of x contains an infinite number of points in X.

(c) x is called a *limit point* of X if there is a sequence $\{x_n : n \in \omega\} \subseteq X$ such that for every open neighborhood N_x of x, there is real number $M > 0$ such that for all $n > M$, $x_n \in N_x$.

In any T_1 space, ($\mathbb{R}$ with the order topology is T_1), a point is a cluster point if and only if it is an accumulation point. (Clearly, a cluster point is an accumulation point. Suppose x is an accumulation point of X that is not a cluster point. Suppose N_x is a neighborhood of x that only contains a finite number of elements of X, x_1, x_2, $\ldots$, x_n. Since X is T_1, using induction, we can find a neighborhood M of x that does not intersect $\{x_1, x_2, \ldots, x_n\}$. Then $N_x \cap M$ is a neighborhood of x that does not contain any points of X different from x. This contradicts the fact that x is an accumulation point.)